



## New Algorithm for Computing Adomian's Polynomials to Solve Coupled Hirota System

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### Abstract

In this paper, when compared to the normal Adomian decomposition approach, we updated the method of calculating Adomian's polynomial to discover the numerical solution for a non-linear coupled Hirota system (CHS) with fewer components, improved accuracy, and faster convergence (ADM). The novel algorithm offers a viable way to computing Adomian polynomials for all types of non-linearity. We can see that these two methods are both effective for solving non-linear CHS, however, the result provided by our new algorithm is superior to that obtained by the classic Adomian decomposition method. Maple 15 was utilized to do calculations in our work.

### Introduction

Non-linear sensations played a major influence in applied mathematics and computational physics. In the existence of computer programming programs, solving a linear equation is not difficult. However, solving non-linear problems analytically remains a challenge for mathematicians. Analytical methods are rapidly evolving, but they still include flaws and inadequacies that do not satisfy mathematicians. Many issues in various disciplines of science can be described using non-linear partial differential equations, as is well known. It's still difficult to solve non-linear models of real-world issues numerically or conceptually. The hunt for better and more effectual solution methods in establishing a solution to non-linear models, whether approximate or exact, analytical or numerical, has gotten a lot of attention [7].

We begin with the Hirota equation [4, 10] to describe non-linear CHS:

$$\frac{\partial w}{\partial t} + 3\alpha|w|^2 \frac{\partial w}{\partial x} + \gamma \frac{\partial^3 w}{\partial x^3} = 0, -\infty < x < \infty, t > 0, \quad (1)$$

where  $w$  is a complex-valued function of the two-dimensional coordinate  $x$  and the time  $\alpha$ , and  $\gamma$  are non-negative real constants. This is an integrable equation having a wide range of physical applications, including the spread of optical pulses in nematic liquid crystal waveguides. The Hirota equation is intricately related to both the non-linear Schrodinger (NLS) and modified Korteweg-de Vries (mKdV) equations since it is a complex generality of the mKdV equation and a member of the NLS ladder of the integrable equation. It has a soliton

solution that is similar to the NLS soliton. With amplitude and velocity, the Hirota equation (1) has a two-parameter soliton domestic. The actual solution of Hirota equation (1) is

$$\begin{aligned}
 w(x, t) &= \beta \operatorname{sech}[k(x - Vt)] \exp(i\varphi), \\
 \beta &= \sqrt{\frac{2\gamma}{\alpha}} k \quad , \quad \varphi = a(x - bt), \\
 V &= \gamma(k^2 - 3a^2) \quad , \quad b = \gamma(3k^2 - a^2).
 \end{aligned}
 \tag{2}$$

The amplitude of the wave is  $w$ , the width of the wave envelope is  $k$ , and the velocity is  $V$ . The parameter  $a$  represents the phase's wavenumber, while  $b$  represents the phase's frequency. Also, the solution is  $x = x_0$  at  $t = 0$ . The Hirota equation has the conserved quantities

$$\begin{aligned}
 I_1 &= \int_{-\infty}^{\infty} |w|^2 dx = \text{constant}, \\
 I_2 &= \int_{-\infty}^{\infty} \left( \frac{\alpha}{2} |w|^4 - |w_x|^2 \right) dx = \text{constant}.
 \end{aligned}
 \tag{3}$$

Wazwaz [16] solved the Hirota equation (1) analytically using the sine-cosine and tanh methods, demonstrating that this equation confesses sech-shaped soliton solutions with free amplitudes and velocities, as well as a tanh solution (kink type). Raslan and Abu Shaer [14] were also solved using the tanh technique. Hirota and Satsuma [9] also employed the Hirota approach for solving problems (1). We presume to avoid difficult computations that need too many calculations in the solution of (1)

$$w(x, t) = u(x, t) + iv(x, t), \quad i^2 = -1$$

where  $u(x, t)$  and  $v(x, t)$  are real functions. This will reduce Hirota equation (1) to the coupled Hirota system (CHS) after computations

$$\begin{aligned}
 \frac{\partial u}{\partial t} + 3\alpha f(u, v) \frac{\partial u}{\partial x} + \gamma \frac{\partial^3 u}{\partial x^3} &= 0, \\
 \frac{\partial v}{\partial t} + 3\alpha f(u, v) \frac{\partial v}{\partial x} + \gamma \frac{\partial^3 v}{\partial x^3} &= 0.
 \end{aligned}
 \tag{4}$$

where  $f(u, v) = u^2 + v^2$ . In this study, we use ADM and NMADM to try to solve (4) numerically. ADM and NMADM are two well-known methods for getting approximate solutions to differential equations, similar to other non-linear analytical approaches [15].

### An Analysis of Adomian's Decomposition Method (ADM)

It's a great way to solve non-linear functional equations. This method is depending on decomposing a non-linear functional equation solution into a series of functions. A polynomial created by a power series expansion of an analytic function yields each term in this series. In theory, these computations are simple, but in fact, computing the polynomials and showing the convergence of the related series can be challenging [1, 2, 3, 6].

Consider the general non-linear coupled of PDEs written in an operator form [5, 8]

$$\begin{aligned}
 L_t(u) + R_1(u) + M_1(u) + N_1(u, v) &= f_1(x, t), \\
 L_t(v) + R_2(v) + M_2(v) + N_2(u, v) &= f_2(x, t),
 \end{aligned}
 \tag{5}$$

subject to initial conditions

$$u(x, 0) = g_1(x), \quad 0 \leq x \leq l_1 \quad \text{and} \quad v(x, 0) = g_2(x), \quad 0 \leq x \leq l_2,$$

where the notations  $L_t = \frac{\partial}{\partial t}$ ,  $R_1$  and  $R_2$  represent the linear spatial differential operators, the notations  $M_1, M_2, N_1$  and  $N_2$  symbolize the non-linear differential operators and  $f_1(x, t), f_2(x, t), (f_1, f_2$  for easiness) are given functions.

**A. The Standard ADM**

Applying the inverse operator,  $L_t^{-1} = \int_0^t (\cdot) dt$  to both sides for Equation (5) provides:

$$\begin{aligned} u(x, t) &= g_1(x) + L_t^{-1}f_1 - L_t^{-1}[R_1(u) + M_1(u) + N_1(u, v)], \\ v(x, t) &= g_2(x) + L_t^{-1}f_2 - L_t^{-1}[R_2(v) + M_2(v) + N_2(u, v)], \end{aligned} \tag{6}$$

which is the main algorithm of ADM. According to the standard ADM, the linear functions  $u(x, t)$  and  $v(x, t)$  are decomposable into an infinite number of components

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), \quad v(x, t) = \sum_{n=0}^{\infty} v_n(x, t),$$

and the non-linear operators  $M_1, M_2, N_1$  and  $N_2$  by the infinite series

$$\begin{aligned} M_1(u) &= \sum_{n=0}^{\infty} A_n, & N_1(u, v) &= \sum_{n=0}^{\infty} B_n, \\ M_2(v) &= \sum_{n=0}^{\infty} C_n, & N_2(u, v) &= \sum_{n=0}^{\infty} D_n, \end{aligned} \tag{7}$$

where  $u_n(x, t)$  and  $v_n(x, t), n = 0, 1, 2, \dots$  are the components of  $u(x, t)$  and  $v(x, t)$  that will be stylistically determined and  $A_n, B_n, C_n$  and  $D_n$  are called Adomian's polynomials. For the non-linear operators  $M_1(u)$  and  $N_1(u, v)$  the Adomian's polynomial can be defined by

$$\begin{aligned} A_n(u_0, u_1, \dots, u_n) &= \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ M_1 \left( \sum_{i=0}^n \lambda^i u_i \right) \right]_{\lambda=0}, \\ B_n(u_0, \dots, u_n; v_0, \dots, v_n) &= \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ N_1 \left( \sum_{i=0}^n \lambda^i u_i, \sum_{i=0}^n \lambda^i v_i \right) \right]_{\lambda=0}. \end{aligned} \tag{8}$$

According to the decomposition method [11,12], the non-linear system (6) is created in a form of the following recursive relations:

$$u_0(x, t) = g_1(x) + L_t^{-1}(f_1), \quad v_0(x, t) = g_2(x) + L_t^{-1}(f_2),$$

and

$$\begin{aligned} u_{n+1}(x, t) &= -L_t^{-1}[R_1(u) + A_n + B_n], \\ v_{n+1}(x, t) &= -L_t^{-1}[R_2(v) + C_n + D_n], \end{aligned} \tag{9}$$

$n = 0, 1, 2, \dots$

It's worth noting that once the zeroth components  $u_0$  and  $v_0$  are defined, the other components  $u_n$  and  $v_n, n \geq 1$  can be entirely determined by computing each component using the preceding terms. As a result, the

components  $u_0, u_1, u_2, \dots$  and  $v_0, v_1, v_2, \dots$  as well as the series solutions, have been identified. For numerical comparison drives, we built the solution  $u(x, t)$ , and  $v(x, t)$  as follows:

$$\lim_{n \rightarrow \infty} \Phi_n(x, t) = u(x, t), \quad \lim_{n \rightarrow \infty} \Psi_n(x, t) = v(x, t),$$

where

$$\Phi_n(x, t) = \sum_{k=0}^n u_k(x, t), \quad \Psi_n(x, t) = \sum_{k=0}^n v_k(x, t), \quad n \geq 0. \quad (10)$$

Furthermore, for a real-world physical problem, the decomposition series solutions converged quickly [13].

### B. New Modification of ADM

We introduce an well-organized novel modification of the Adomian decomposition method (NMADM), which decomposes the non-linear term into two parts by collecting all terms with the same sum of subscripts of the components  $u_n$ , the term for more than time in computing the polynomials. So, the polynomials for the non-linear term  $M_1(u) = \bar{M}_1(m_1, m_2)$  can be obtained as follows [13]:

$$\bar{A}_0(u_0) = \bar{M}_1(m_{1_0}, m_{2_0}),$$

$$\bar{A}_1(u_0, u_1) = \bar{M}_1(m_{1_0}, m_{2_1}) + \bar{M}_1(m_{1_1}, m_{2_0}),$$

$$\bar{A}_2(u_0, u_1, u_2) = \bar{M}_1(m_{1_0}, m_{2_2}) + \bar{M}_1(m_{1_2}, m_{2_0}) + \bar{M}_1(m_{1_1}, m_{2_1}),$$

⋮

The  $\bar{A}_n$  can be lastly written as the following suitable relation

$$\bar{A}_i = \sum_{j=i}^{2i} \rho^j (\bar{M}_1(m_1, m_2)) = \sum_{\substack{j=i \\ k+h=j}}^{2i} \bar{M}_1(m_{1_k}, m_{2_h}), \quad i, k, h = 0, 1, 2, \dots \quad (11)$$

where  $\rho$  is decomposed of the non-linear term and  $m_1$  and  $m_2$  represent the dependent variable  $u$ . The polynomials for the non-linear term  $N_1(u, v) = \bar{N}_1(n_1, n_2)$  are obtained using the same procedure:

$$\bar{B}_0(u_0; v_0) = \bar{N}_1(n_{1_0}, n_{2_0}),$$

$$\bar{B}_1(u_0, u_1; v_0, v_1) = \bar{N}_1(n_{1_0}, n_{2_1}) + \bar{N}_1(n_{1_1}, n_{2_0}),$$

$$\bar{B}_2(u_0, u_1, u_2; v_0, v_1, v_2) = \bar{N}_1(n_{1_0}, n_{2_2}) + \bar{N}_1(n_{1_2}, n_{2_0}) + \bar{N}_1(n_{1_1}, n_{2_1}),$$

⋮

Finally, the  $\bar{B}_n$  can be expressed as the following useful formula

$$\bar{B}_i = \sum_{k+h=i} \bar{N}_1(n_{1_k}, n_{2_h}), \quad i, k, h = 0, 1, 2, \dots \quad (12)$$

where  $n_1$  and  $n_2$  signify the dependent variable  $u$  or  $v$ , and so on, likewise for finding the polynomials  $\bar{C}_n$  and  $\bar{D}_n$ .

### Reformulation of ADM to Solve the CHS

To apply the standard ADM on the CHS, we construct the following:

We use Equation (2) to find ADM of the CHS which becomes:

$$u(x, t) = u(x, 0) + L_t^{-1} \left[ -3\alpha u^2 \frac{\partial u}{\partial x} - 3\alpha v^2 \frac{\partial u}{\partial x} - \gamma \frac{\partial^3 u}{\partial x^3} \right],$$

and

$$v(x, t) = v(x, 0) + L_t^{-1} \left[ -3\alpha u^2 \frac{\partial v}{\partial x} - 3\alpha v^2 \frac{\partial v}{\partial x} - \gamma \frac{\partial^3 v}{\partial x^3} \right].$$

and then by series (7), we obtain:

$$u(x, t) = u(x, 0) + L_t^{-1} \left[ -3\alpha \sum_{n=0}^{\infty} A_n - 3\alpha \sum_{n=0}^{\infty} B_n - \gamma \frac{\partial^3 u_n}{\partial x^3} \right],$$

$$v(x, t) = v(x, 0) + L_t^{-1} \left[ -3\alpha \sum_{n=0}^{\infty} C_n - 3\alpha \sum_{n=0}^{\infty} D_n - \gamma \frac{\partial^3 v_n}{\partial x^3} \right].$$

where

$$u^2 \frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} A_n, \quad v^2 \frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} B_n,$$

$$v^2 \frac{\partial v}{\partial x} = \sum_{n=0}^{\infty} C_n, \quad u^2 \frac{\partial v}{\partial x} = \sum_{n=0}^{\infty} D_n.$$

The Adomian's polynomials  $A_n, B_n, C_n$  and  $D_n$  are made according to (8) we can give the first few Adomian's polynomials of  $A_n$  and  $B_n$  correspondingly:

$$A_0 = u_0^2 \frac{\partial u_0}{\partial x},$$

$$A_1 = u_0^2 \frac{\partial u_1}{\partial x} + 2u_0 u_1 \frac{\partial u_0}{\partial x},$$

$$A_2 = u_0^2 \frac{\partial u_2}{\partial x} + 2u_0 u_1 \frac{\partial u_1}{\partial x} + 2u_0 u_2 \frac{\partial u_0}{\partial x} + u_1^2 \frac{\partial u_0}{\partial x},$$

$$A_3 = u_0^2 \frac{\partial u_3}{\partial x} + 2u_0 u_1 \frac{\partial u_2}{\partial x} + u_1^2 \frac{\partial u_1}{\partial x} + 2u_0 u_3 \frac{\partial u_0}{\partial x} + 2u_1 u_2 \frac{\partial u_0}{\partial x},$$

$$\vdots$$

$$B_0 = v_0^2 \frac{\partial u_0}{\partial x},$$

$$B_1 = v_0^2 \frac{\partial u_1}{\partial x} + 2v_0 v_1 \frac{\partial u_0}{\partial x},$$

$$\begin{aligned}
 B_2 &= v_0^2 \frac{\partial u_2}{\partial x} + 2v_0v_1 \frac{\partial u_1}{\partial x} + 2v_0v_2 \frac{\partial u_0}{\partial x} + v_1^2 \frac{\partial u_0}{\partial x}, \\
 B_3 &= v_0^2 \frac{\partial u_3}{\partial x} + 2v_0v_1 \frac{\partial u_2}{\partial x} + v_1^2 \frac{\partial u_1}{\partial x} + 2v_0v_3 \frac{\partial u_0}{\partial x} + 2v_1v_2 \frac{\partial u_0}{\partial x}. \\
 &\vdots
 \end{aligned}$$

The polynomials  $C_n$  and  $D_n$  can be produced in the same way as the polynomials  $C_n$  and  $D_n$  in the preceding approach. The zeroth components  $u_0$  and  $v_0$  are written as follows according to (9):

$$\begin{aligned}
 u_0(x, t) &= u(x, 0), \\
 v_0(x, t) &= v(x, 0).
 \end{aligned}$$

and the recursive relation can be written as follows:

$$u_{n+1}(x, t) = L_t^{-1} \left[ -3\alpha A_n - 3\alpha B_n - \gamma \frac{\partial^3 u_n}{\partial x^3} \right], \tag{13}$$

and

$$v_{n+1}(x, t) = L_t^{-1} \left[ -3\alpha C_n - 3\alpha D_n - \gamma \frac{\partial^3 v_n}{\partial x^3} \right]. \tag{14}$$

where  $n = 0, 1, 2, \dots$ . So, we get the following components:

Now, From the Equations (13)-(14), put  $n = 0$ , we have:

$$\begin{aligned}
 u_1(x, t) &= L_t^{-1} \left[ -3\alpha A_0 - 3\alpha B_0 - \gamma \frac{\partial^3 u_0}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha u_0^2 \frac{\partial u_0}{\partial x} - 3\alpha v_0^2 \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right] \\
 &= \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right) t \\
 &= w_{10}(x, t)t,
 \end{aligned} \tag{15}$$

and

$$\begin{aligned}
 v_1(x, t) &= L_t^{-1} \left[ -3\alpha C_0 - 3\alpha D_0 - \gamma \frac{\partial^3 v_0}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha v_0^2 \frac{\partial v_0}{\partial x} - 3\alpha u_0^2 \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right] \\
 &= \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right) t \\
 &= w_{20}(x, t)t,
 \end{aligned} \tag{16}$$

where

$$w_{10}(x, t) = \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right),$$

and

$$w_{20}(x, t) = \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right).$$

From the Equations (13)-(14), put  $n = 1$ , we have:

$$\begin{aligned} u_2(x, t) &= L_t^{-1} \left[ -3\alpha A_1 - 3\alpha B_1 - \gamma \frac{\partial^3 u_1}{\partial x^3} \right] \\ &= L_t^{-1} \left[ -3\alpha \left( 2u_0 u_1 \frac{\partial u_0}{\partial x} + u_0^2 \frac{\partial u_1}{\partial x} \right) - 3\alpha \left( 2v_0 v_1 \frac{\partial u_0}{\partial x} + v_0^2 \frac{\partial u_1}{\partial x} \right) - \gamma \frac{\partial^3 u_1}{\partial x^3} \right] \\ &= L_t^{-1} \left[ -6\alpha(u_0 u_1 + v_0 v_1) \frac{\partial u_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial u_1}{\partial x} - \gamma \frac{\partial^3 u_1}{\partial x^3} \right], \end{aligned} \tag{17}$$

and

$$\begin{aligned} v_2(x, t) &= L_t^{-1} \left[ -3\alpha C_1 - 3\alpha D_1 - \gamma \frac{\partial^3 v_1}{\partial x^3} \right] \\ &= L_t^{-1} \left[ -3\alpha \left( 2v_0 v_1 \frac{\partial v_0}{\partial x} + v_0^2 \frac{\partial v_1}{\partial x} \right) - 3\alpha \left( 2u_0 u_1 \frac{\partial v_0}{\partial x} + u_0^2 \frac{\partial v_1}{\partial x} \right) - \gamma \frac{\partial^3 v_1}{\partial x^3} \right] \\ &= L_t^{-1} \left[ -6\alpha(u_0 u_1 + v_0 v_1) \frac{\partial v_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial v_1}{\partial x} - \gamma \frac{\partial^3 v_1}{\partial x^3} \right]. \end{aligned} \tag{18}$$

Substituting Equations (15)-(16), into Equations (17)-(18), we get:

$$\begin{aligned} u_2(x, t) &= L_t^{-1} \left[ -6\alpha t(u_0 w_{10} + v_0 w_{20}) \frac{\partial u_0}{\partial x} - 3\alpha t(u_0^2 + v_0^2) \frac{\partial w_{10}}{\partial x} - \gamma t \frac{\partial^3 w_{10}}{\partial x^3} \right] \\ &= \left( -6\alpha(u_0 w_{10} + v_0 w_{20}) \frac{\partial u_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial w_{10}}{\partial x} - \gamma \frac{\partial^3 w_{10}}{\partial x^3} \right) \left( \frac{t^2}{2} \right) \\ &= w_{11}(x, t) \left( \frac{t^2}{2} \right), \end{aligned}$$

and

$$\begin{aligned} v_2(x, t) &= L_t^{-1} \left[ -6\alpha t(u_0 w_{10} + v_0 w_{20}) \frac{\partial v_0}{\partial x} - 3\alpha t(u_0^2 + v_0^2) \frac{\partial w_{20}}{\partial x} - \gamma t \frac{\partial^3 w_{20}}{\partial x^3} \right] \\ &= \left( -6\alpha(u_0 w_{10} + v_0 w_{20}) \frac{\partial v_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial w_{20}}{\partial x} - \gamma \frac{\partial^3 w_{20}}{\partial x^3} \right) \left( \frac{t^2}{2} \right) \\ &= w_{21}(x, t) \left( \frac{t^2}{2} \right), \end{aligned}$$

where

$$w_{11}(x, t) = \left( -6\alpha(u_0 w_{10} + v_0 w_{20}) \frac{\partial u_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial w_{10}}{\partial x} - \gamma \frac{\partial^3 w_{10}}{\partial x^3} \right),$$

and

$$w_{21}(x, t) = \left( -6\alpha(u_0 w_{10} + v_0 w_{20}) \frac{\partial v_0}{\partial x} - 3\alpha(u_0^2 + v_0^2) \frac{\partial w_{20}}{\partial x} - \gamma \frac{\partial^3 w_{20}}{\partial x^3} \right).$$

We get closed-form exact solutions by substituting all components in decomposition series (10) which is a Taylor series.

### Reformulation of NMADM to Solve the CHS

To apply the new modification to ADM on the CHS, we construct the following:

We use Equation (6) to find NADM of the CHS which becomes:

$$u(x, t) = u(x, 0) + L_t^{-1} \left[ -3\alpha u^2 \frac{\partial u}{\partial x} - 3\alpha v^2 \frac{\partial u}{\partial x} - \gamma \frac{\partial^3 u}{\partial x^3} \right],$$

and

$$v(x, t) = v(x, 0) + L_t^{-1} \left[ -3\alpha u^2 \frac{\partial v}{\partial x} - 3\alpha v^2 \frac{\partial v}{\partial x} - \gamma \frac{\partial^3 v}{\partial x^3} \right].$$

and then by series (7), we obtain:

$$u(x, t) = u(x, 0) + L_t^{-1} \left[ -3\alpha \sum_{n=0}^{\infty} A_n - 3\alpha \sum_{n=0}^{\infty} B_n - \gamma \frac{\partial^3 u_n}{\partial x^3} \right],$$

$$v(x, t) = v(x, 0) + L_t^{-1} \left[ -3\alpha \sum_{n=0}^{\infty} C_n - 3\alpha \sum_{n=0}^{\infty} D_n - \gamma \frac{\partial^3 v_n}{\partial x^3} \right].$$

where

$$u^2 \frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} A_n, \quad v^2 \frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} B_n,$$

$$v^2 \frac{\partial v}{\partial x} = \sum_{n=0}^{\infty} C_n, \quad u^2 \frac{\partial v}{\partial x} = \sum_{n=0}^{\infty} D_n.$$

Now, if we use the new method (11)-(12) for non-linear operators  $M_1$  and  $N_1$  we will get the following form of  $\bar{A}_n$  and  $\bar{B}_n$  respectively:

$$M_1(u) = u^2 \frac{\partial u}{\partial x} = \bar{M}_1(m_1, m_2),$$

$$\bar{A}_0 = u_0^2 \frac{\partial u_0}{\partial x},$$

$$\begin{aligned} \bar{A}_1 &= u_0^2 \frac{\partial u_1}{\partial x} + 2u_0u_1 \frac{\partial u_0}{\partial x} + u_1^2 \frac{\partial u_0}{\partial x}, \\ \bar{A}_2 &= u_0^2 \frac{\partial u_2}{\partial x} + 2u_0u_1 \frac{\partial u_1}{\partial x} + 2u_0u_2 \frac{\partial u_0}{\partial x} + u_1^2 \frac{\partial u_1}{\partial x} + u_2^2 \frac{\partial u_0}{\partial x}, \\ \bar{A}_3 &= u_0^2 \frac{\partial u_3}{\partial x} + 2u_0u_1 \frac{\partial u_2}{\partial x} + 2u_0u_3 \frac{\partial u_0}{\partial x} + 2u_1u_2 \frac{\partial u_0}{\partial x} + 2u_0u_2 \frac{\partial u_1}{\partial x} + u_1^2 \frac{\partial u_2}{\partial x} + u_2^2 \frac{\partial u_1}{\partial x} + u_3^2 \frac{\partial u_0}{\partial x}, \\ &\vdots \\ N_1(u, v) &= v^2 \frac{\partial u}{\partial x} = \bar{N}_1(u, v), \\ \bar{B}_0 &= v_0^2 \frac{\partial u_0}{\partial x}, \\ \bar{B}_1 &= v_0^2 \frac{\partial u_1}{\partial x} + 2v_0v_1 \frac{\partial u_0}{\partial x} + v_1^2 \frac{\partial u_0}{\partial x}, \\ \bar{B}_2 &= v_0^2 \frac{\partial u_2}{\partial x} + 2v_0v_1 \frac{\partial u_1}{\partial x} + 2v_0v_2 \frac{\partial u_0}{\partial x} + v_1^2 \frac{\partial u_1}{\partial x} + v_2^2 \frac{\partial u_0}{\partial x}, \\ \bar{B}_3 &= v_0^2 \frac{\partial u_3}{\partial x} + 2v_0v_1 \frac{\partial u_2}{\partial x} + 2v_0v_3 \frac{\partial u_0}{\partial x} + 2v_1v_2 \frac{\partial u_0}{\partial x} + 2v_0v_2 \frac{\partial u_1}{\partial x} + v_1^2 \frac{\partial u_2}{\partial x} + v_2^2 \frac{\partial u_1}{\partial x} + v_3^2 \frac{\partial u_0}{\partial x}. \\ &\vdots \end{aligned}$$

The polynomials  $\bar{C}_n$  and  $\bar{D}_n$  can be formed in the same way as the polynomials  $\bar{C}_n$  and  $\bar{D}_n$  in the preceding approach. The zeroth components  $u_0$  and  $v_0$  are written as follows according to (9):

$$\begin{aligned} u_0(x, t) &= u_0, \\ v_0(x, t) &= v_0. \end{aligned}$$

and the recursive relation can be written as follows:

$$u_{n+1}(x, t) = L_t^{-1} \left[ -3\alpha\bar{A}_n - 3\alpha\bar{B}_n - \gamma \frac{\partial^3 u_n}{\partial x^3} \right], \tag{19}$$

and

$$v_{n+1}(x, t) = L_t^{-1} \left[ -3\alpha\bar{C}_n - 3\alpha\bar{D}_n - \gamma \frac{\partial^3 v_n}{\partial x^3} \right]. \tag{20}$$

where  $n = 0, 1, 2, \dots$ . So, we get the following components:

Now, From the Equations (19)-(20), put  $n = 0$ , we have:

$$\begin{aligned} u_1(x, t) &= L_t^{-1} \left[ -3\alpha\bar{A}_0 - 3\alpha\bar{B}_0 - \gamma \frac{\partial^3 u_0}{\partial x^3} \right] \\ &= L_t^{-1} \left[ -3\alpha u_0^2 \frac{\partial u_0}{\partial x} - 3\alpha v_0^2 \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right] \end{aligned}$$

$$\begin{aligned}
 &= \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right) t \\
 &= w_{10}(x, t)t,
 \end{aligned} \tag{21}$$

and

$$\begin{aligned}
 v_1(x, t) &= L_t^{-1} \left[ -3\alpha\bar{C}_0 - 3\alpha\bar{D}_0 - \gamma \frac{\partial^3 v_0}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha u_0^2 \frac{\partial v_0}{\partial x} - 3\alpha v_0^2 \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right] \\
 &= \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right) t \\
 &= w_{20}(x, t)t,
 \end{aligned} \tag{22}$$

where

$$w_{10}(x, t) = \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_0}{\partial x^3} \right),$$

and

$$w_{20}(x, t) = \left( -3\alpha(u_0^2 + v_0^2) \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_0}{\partial x^3} \right).$$

From the Equations (19)-(20), put  $n = 1$ , we have:

$$\begin{aligned}
 u_2(x, t) &= L_t^{-1} \left[ -3\alpha\bar{A}_1 - 3\alpha\bar{B}_1 - \gamma \frac{\partial^3 u_1}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha \left( u_0^2 \frac{\partial u_1}{\partial x} + 2u_0 u_1 \frac{\partial u_0}{\partial x} + u_1^2 \frac{\partial u_0}{\partial x} \right) - 3\alpha \left( v_0^2 \frac{\partial u_1}{\partial x} + 2v_0 v_1 \frac{\partial u_0}{\partial x} + v_1^2 \frac{\partial u_0}{\partial x} \right) - \gamma \frac{\partial^3 u_1}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha(u_0^2 + v_0^2) \frac{\partial u_1}{\partial x} - 6\alpha(v_0 v_1 + u_0 u_1) \frac{\partial u_0}{\partial x} - 3\alpha(u_1^2 + v_1^2) \frac{\partial u_0}{\partial x} - \gamma \frac{\partial^3 u_1}{\partial x^3} \right] \\
 &= w_{11}(x, t) \left( \frac{t^2}{2} \right),
 \end{aligned}$$

and

$$\begin{aligned}
 v_2(x, t) &= L_t^{-1} \left[ -3\alpha\bar{C}_1 - 3\alpha\bar{D}_1 - \gamma \frac{\partial^3 v_1}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha \left( u_0^2 \frac{\partial v_1}{\partial x} + 2u_0 u_1 \frac{\partial v_0}{\partial x} + u_1^2 \frac{\partial v_0}{\partial x} \right) - 3\alpha \left( v_0^2 \frac{\partial v_1}{\partial x} + 2v_0 v_1 \frac{\partial v_0}{\partial x} + v_1^2 \frac{\partial v_0}{\partial x} \right) - \gamma \frac{\partial^3 v_1}{\partial x^3} \right] \\
 &= L_t^{-1} \left[ -3\alpha(u_0^2 + v_0^2) \frac{\partial v_1}{\partial x} - 6\alpha(v_0 v_1 + u_0 u_1) \frac{\partial v_0}{\partial x} - 3\alpha(u_1^2 + v_1^2) \frac{\partial v_0}{\partial x} - \gamma \frac{\partial^3 v_1}{\partial x^3} \right]
 \end{aligned}$$

$$= w_{21}(x, t) \left( \frac{t^2}{2} \right),$$

where

$$w_{11}(x, t) = -3\alpha \left( (u_0^2 + v_0^2) \frac{\partial u_1}{\partial x} + 2(v_0 v_1 + u_0 u_1) \frac{\partial u_0}{\partial x} + (u_1^2 + v_1^2) \frac{\partial u_0}{\partial x} \right) - \gamma \frac{\partial^3 u_1}{\partial x^3},$$

and

$$w_{21}(x, t) = -3\alpha \left( (u_0^2 + v_0^2) \frac{\partial v_1}{\partial x} + 2(v_0 v_1 + u_0 u_1) \frac{\partial v_0}{\partial x} + (u_1^2 + v_1^2) \frac{\partial v_0}{\partial x} \right) - \gamma \frac{\partial^3 v_1}{\partial x^3}.$$

We get closed-form exact solutions by substituting all components in decomposition series (10) which is a Taylor series.

### Numerical Applications

**Example:** Consider the following non-linear CHS:

$$\begin{aligned} \frac{\partial u}{\partial t} + 3\alpha(u^2 + v^2) \frac{\partial u}{\partial x} + \gamma \frac{\partial^3 u}{\partial x^3} &= 0, \\ \frac{\partial v}{\partial t} + 3\alpha(u^2 + v^2) \frac{\partial v}{\partial x} + \gamma \frac{\partial^3 v}{\partial x^3} &= 0. \end{aligned}$$

with the initial conditions

$$u_0(x, t) = \sqrt{\frac{2\gamma}{\alpha}} k \operatorname{sech}(kx) \cos(ax),$$

and

$$v_0(x, t) = \sqrt{\frac{2\gamma}{\alpha}} k \operatorname{sech}(kx) \sin(ax),$$

where  $k$  and  $a$  are arbitrary constants.

The exact solutions of the Hirota equation (1) are given by (2), where  $\alpha$  and  $\gamma > 0$ , are arbitrary constants.

**Solution:**

**First using ADM:**

Apply the ADM (see: Reformulation of ADM to Solve the CHS), we get the following components:

$$u_1 = \frac{-k\gamma^{\frac{3}{2}} \sqrt{\frac{2}{\alpha}} t}{\cosh(kx)^2} \left( (-k^3 + 3a^2 k) \sinh(kx) \cos(ax) + (a^3 - 3a k^2) \cosh(kx) \sin(ax) \right),$$

$$v_1 = \frac{k\gamma^{\frac{3}{2}} \sqrt{\frac{2}{\alpha}} t}{\cosh(kx)^2} \left( (k^3 - 3a^2 k) \sinh(kx) \sin(ax) + (a^3 - 3a k^2) \cosh(kx) \cos(ax) \right),$$

$$u_2 = \frac{-k\gamma^{\frac{5}{2}} \sqrt{\frac{2}{\alpha}} t^2}{2\cosh(kx)^3} \left( (2k^6 - 12a^2 k^4 + 18a^4 k^2) \cos(ax) \right.$$

$$\left. + (15a^2 k^4 - 15a^4 k^2 - k^6 + a^6) \cosh(kx)^2 \cos(ax) \right)$$

$$\begin{aligned}
 & + (20a^3k^3 - 6ak^5 - 6a^5k) \sinh(kx) \cosh(kx) \sin(ax)), \\
 v_2 = & \frac{-k \gamma^{\frac{5}{2}} \sqrt{\frac{2}{\alpha}} t^2}{2 \cosh(kx)^3} ((2k^6 - 12a^2 k^4 + 18a^4 k^2) \sin(ax) \\
 & + (a^6 - 15a^4k^2 + 15a^2k^4 - k^6) \cosh(ax)^2 \sin(ax) \\
 & + (6a k^5 - 20a^3k^3 + 6a^5 k) \sinh(kx) \cosh(kx) \cos(ax)), \\
 & \vdots
 \end{aligned}$$

**Second, using NMADM:**

Applying NMADM (see: Reformulation of NMADM to Solve the CHS) by utilizing (9)- (10), it's significant that the zeroth and first segments are comparable in two standard ADM equations, yet different parts are unique; therefore, the inexact arrangement has higher precision and quicker union to the specific arrangement than the standard ADM, where different segments can be composed as follows:

$$\begin{aligned}
 u_1 = & \frac{-k \gamma^{\frac{3}{2}} \sqrt{\frac{2}{\alpha}} t}{\cosh(kx)^2} ((-k^3 + 3a^2k) \sinh(kx) \cos(ax) \\
 & + (a^3 - 3a k^2) \cosh(kx) \sin(ax)), \\
 v_1 = & \frac{k \gamma^{\frac{3}{2}} \sqrt{\frac{2}{\alpha}} t}{\cosh(kx)^2} ((k^3 - 3a^2k) \sinh(kx) \sin(ax) \\
 & + (a^3 - 3ak^2) \cosh(kx) \cos(ax)), \\
 u_2 = & \frac{k \gamma^{\frac{5}{2}} \sqrt{\frac{2}{\alpha}} t^2}{2 \cosh(kx)^6} ((-20a^3k^3 + a^5k + 15 a k^5) \sinh(kx) \cosh(kx)^4 \sin(ax) \\
 & + (4\gamma ak^8 + 12\gamma ak^4 - 4\gamma k^9 + 12\gamma ak^6 + 4\gamma a^7k^2)t \cosh(kx)^3 \sin(ax) \\
 & + k^6 \cosh(kx)^4 \sinh(kx) + k^6 \cosh(kx)^5 \cos(ax) - a^6 \cosh(kx) \cos(ax) \\
 & + (12a^2 k^7 + 12a^4 k^5 + 4a^6k^3)\gamma t \sinh(kx) \cosh(kx)^2 \cos(ax) \\
 & + (24a^3k^6 - 36a^5k^4 - 4a k^8)\gamma t \cosh(kx) \sin(ax) \\
 & + (12a^2 k^4 - 2k^6 - 18a^4 k^2) \cosh(kx)^3 \cos(ax) \\
 & + (24 a^2 k^7 - 36a k^5)\gamma t \sinh(kx) \cos(ax) \\
 & + 4\gamma k^9 t \sinh(kx) \cosh(kx)^2 \cos(ax)), \\
 v_2 = & \frac{-k \gamma^{\frac{5}{2}} \sqrt{\frac{2}{\alpha}} t^2}{2 \cosh(kx)^2} ((4 \gamma k^9 - 24 \gamma a^2k^7 + 36 \gamma a^4 k^5)t \sinh(kx) \sin(ax) \\
 & - (4a^6k^3 + 12a^4k^5 + 12a k^7 + 4k^9)\gamma t \sinh(kx) \cosh(kx)^2 \sin(ax)
 \end{aligned}$$

$$\begin{aligned}
 &+(12a^5 k^4 + 4a^7 k^2 + 12a^3 k^6 + 4ak^8)\gamma t \cosh(kx)^3 \cos(ax) \\
 &+(6a^5 k - 20a^3 k^3 + 6ak^5) \sinh(kx) \cosh(kx)^4 \cos(ax) \\
 &+15a^2 k^4 \cosh(kx) \sin(ax) - 15a^4 k^2 \cosh(kx) \sin(ax) \\
 &+(24a^3 k^6 - 4a k^8 - 36a^5 k^4)\gamma t \cosh(kx) \cos(ax) \\
 &-k^6 \cosh(kx)^5 \sin(ax) + a^6 \cosh(kx)^2 \sin(ax) \\
 &+(18a^4 k^2 - 12a^2 k^4 + 2k^6) \cosh(kx)^3 \sin(ax), \\
 &\vdots
 \end{aligned}$$

We acquire the following numerical solutions of  $u(x, t)$ , and  $v(x, t)$  by inserting the above components in the decomposition series (10), which gives us the exact closed-form solutions.

**Note:**

The results obtained by ADM and NMADM for the above example, are tabulated in the tables listed below followed by their figures for  $\alpha = 2$ ,  $\gamma = 1$ ,  $k = 0.2$ , and  $a = 0.5$ .

**Table-1:** Comparison of the exact solution of  $u(x, t)$  with the approximate solution obtained by ADM and NMADM,  $-1 \leq x \leq 1$  and  $0 \leq t \leq 1$ .

Time $t$	Exact Solution	ADM	NMADM
<b><math>x = -1</math></b>			
0	0.1720637511	0.1720637511	0.1720637511
0.2	0.1741773912	0.1741785595	0.1741783937
0.4	0.1761408416	0.1761500219	0.1761486957
0.6	0.1779477417	0.1779781382	0.1779736622
0.8	0.1795923046	0.1796629085	0.1796522989
1	0.1810693644	0.1812043327	0.1811836107
<b><math>x = -0.6</math></b>			
0	0.1896998193	0.1896998193	0.1896998193
0.2	0.1910175512	0.1910183222	0.1910182258
0.4	0.1921553676	0.1921613008	0.1921605294
0.6	0.1931095394	0.1931287553	0.1931261518
0.8	0.1938770877	0.1939206856	0.1939145142
1	0.1944558081	0.1945370916	0.1945250381
<b><math>x = -0.2</math></b>			
0	0.1988417385	0.1988417385	0.1988417385
0.2	0.1992301729	0.1992304215	0.1992303900
0.4	0.1994245064	0.1994262240	0.1994259723
0.6	0.1994242648	0.1994291461	0.1994282967
0.8	0.1992297907	0.1992391878	0.1992371744
1	0.1988422381	0.1988563490	0.1988524164
<b><math>x = 0</math></b>			
0	0.2	0.2	0.2
0.2	0.1999024782	0.199902444	0.199902444
0.4	0.1996103218	0.199609776	0.199609776
0.6	0.1991247547	0.199121996	0.199121996
0.8	0.1984478028	0.198439104	0.198439104
1	0.1975822747	0.197561100	0.197561100
<b><math>x = 0.2</math></b>			
0	0.1988417385	0.1988417385	0.1988417385
0.2	0.1982604905	0.1982601751	0.1982602065
0.4	0.1974885191	0.1974857312	0.1974859829
0.6	0.1965286982	0.1965184069	0.1965192564
0.8	0.1953846585	0.1953582022	0.1953602157
1	0.1940607552	0.1940051172	0.1940090498
<b><math>x = 0.6</math></b>			
0	0.1896998193	0.1896998193	0.1896998193
0.2	0.1882066201	0.1882057922	0.1882058886
0.4	0.1865430822	0.1865362410	0.1865370124
0.6	0.1847149709	0.1846911655	0.1846937691
0.8	0.1827286388	0.1826705658	0.1826767372
1	0.1805909717	0.1804744420	0.1804864955
<b><math>x = 1</math></b>			
0	0.1720637511	0.1720637511	0.1720637511
0.2	0.1698068039	0.1698055967	0.1698057625
0.4	0.1674138977	0.1674040961	0.1674054223
0.6	0.1648927880	0.1648592496	0.1648637255
0.8	0.1622515760	0.1621710569	0.1621816666
1	0.1594986469	0.1593395183	0.1593602403

**Table-2:** Comparison of the exact solution of  $v(x, t)$  with the approximate solution obtained by ADM and NMADM,  $-1 \leq x \leq 1$  and  $0 \leq t \leq 1$ .

Time $t$	Exact Solution	ADM	NMADM
<b><math>x = -1</math></b>			
0	-0.09399885566	-0.09399885566	-0.09399885566
0.2	-0.09223403206	-0.09223349798	-0.09223375168
0.4	-0.09036171140	-0.09035726897	-0.09035929856
0.6	-0.08838572802	-0.08837016865	-0.08837701852
0.8	-0.08631040194	-0.08627219701	-0.08628843373
1	-0.08414051626	-0.08406335405	-0.08409506640
<b><math>x = -0.6</math></b>			
0	-0.05868103068	-0.05868103068	-0.05868103068
0.2	-0.05637855296	-0.05637765083	-0.05637791686
0.4	-0.05400897690	-0.05400164552	-0.05400377380
0.6	-0.05157812790	-0.05155301477	-0.05156019772
0.8	-0.04909212276	-0.04903175856	-0.04904878481
1	-0.04655732978	-0.04643787691	-0.04647113130
<b><math>x = -0.2</math></b>			
0	-0.019950720630	-0.019950720630	-0.019950720630
0.2	-0.017376889080	-0.017375789200	-0.017376059210
0.4	-0.014784409810	-0.014775576220	-0.014777736360
0.6	-0.012179991150	-0.012150081690	-0.012157372170
0.8	-0.009570381134	-0.009499305618	-0.009516586756
1	-0.006962321512	-0.006823247998	-0.006857000222
<b><math>x = 0</math></b>			
0	0.00	0.00	0.00
0.2	0.002598878622	0.00260000000	0.00259972960
0.4	0.005191038134	0.00520000000	0.00519783680
0.6	0.007769805124	0.00780000000	0.00779269920
0.8	0.010328596930	0.01040000000	0.01038269440
1	0.012860965490	0.01300000000	0.01296620000
<b><math>x = 0.2</math></b>			
0	0.01995072063	0.01995072063	0.01995072063
0.2	0.02249928150	0.02250037052	0.02250010050
0.4	0.02501607882	0.02502473886	0.02502257871
0.6	0.02749479318	0.02752382565	0.02751653517
0.8	0.02992931956	0.02999763090	0.02998034976
1	0.03231380604	0.03244615460	0.03241240238
<b><math>x = 0.6</math></b>			
0	0.05868103068	0.05868103068	0.05868103068
0.2	0.06091091310	0.06091178509	0.06091151905
0.4	0.06306306508	0.06306991404	0.06306778576
0.6	0.06513274224	0.06515541755	0.06514823460
0.8	0.06711561602	0.06716829560	0.06715126936
1	0.06900779384	0.06910854821	0.06907529382
<b><math>x = 1</math></b>			
0	0.09399885566	0.09399885566	0.09399885566
0.2	0.09565285094	0.09565334202	0.09565308832
0.4	0.09719320186	0.09719695707	0.09719492748
0.6	0.09861761458	0.09862970079	0.09862285093
0.8	0.09992431910	0.09995157325	0.09993533652
1	0.10111206590	0.10116257430	0.10113086200

**Table-3:** Least Square Error for the results in Table 1.

Space $x$	Least Square Error for ADM	Least Square Error for NMADM
-1	2.42109E-08	1.73861E-08
-0.6	8.91283E-09	6.49661E-09
-0.2	3.14262E-10	1.76569E-10
0	5.31947E-10	5.31947E-10
0.2	3.90931E-09	3.36656E-09
0.6	1.75658E-08	1.40959E-08
1	3.30276E-08	2.49613E-08

**Table-4:** Least Square Error for the results in Table 2.

Space $x$	Least Square Error for ADM	Least Square Error for NMADM
-1	7.6757E-09	2.63005E-09
-0.6	1.85981E-08	9.65732E-09
-0.2	2.5367E-08	1.45433E-08
0	2.54223E-08	1.45719E-08
0.2	2.31016E-08	1.2841E-08
0.6	1.34884E-08	6.09007E-09
1	3.45431E-09	5.0513E-10

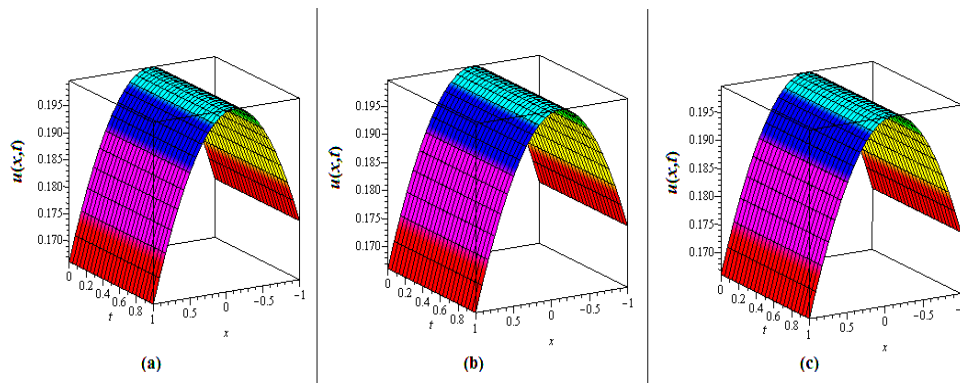


Figure-1: Plots of results of example 1, when  $-1 \leq x \leq 1$ ,  $0 \leq t \leq 1$ ,  $\alpha = 2$ ,  $\gamma = 1$ ,  $k = 0.2$ , and  $a = 0.5$ . (a) Exact solution of  $u(x, t)$ , (b) Approximate solution of  $u(x, t)$  by ADM, (c) Approximate solution of  $u(x, t)$  by NMADM.

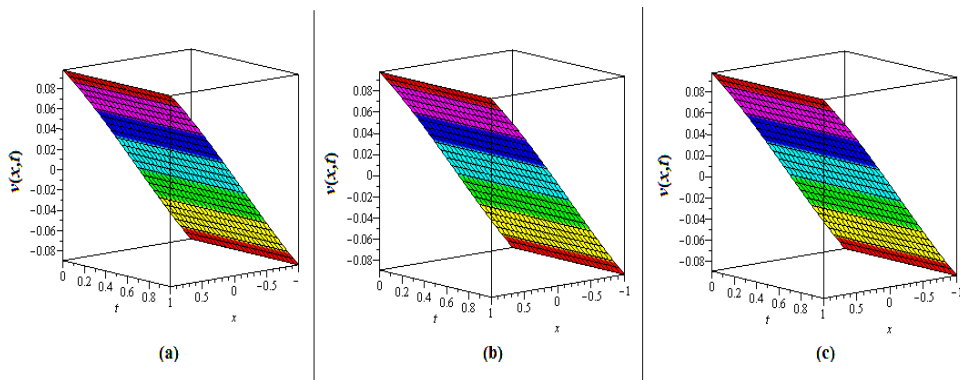


Figure-2: Plots of results of Example 5.1, when  $-1 \leq x \leq 1$ ,  $0 \leq t \leq 1$ ,  $\alpha = 2$ ,  $\gamma = 1$ ,  $k = 0.2$  and  $a = 0.5$ . (a) Exact solution of  $v(x, t)$ , (b) Approximate solution of  $v(x, t)$  by ADM, (c) Approximate solution of  $v(x, t)$  by NMADM.

## Conclusion

In this paper, we show that our technique for calculating Adomian polynomials is more efficient than existing Adomian methods for calculating Adomian polynomials. Tables (1)-(4) show that our CHS solution technique yields fewer errors in most circumstances than the usual Adomian method, we note that in most cases our CHS solution algorithm produces less square errors than the standard Adomian method.

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